INTERACTION OF RADIATION AND CONVECTION IN COMBINED HEAT TRANSFER IN A SHORT CHANNEL

V. I. Shcherbinin, A. Kh. Bokovikova, and F. R. Shklyar

UDC 536.3:536.25

The composite heat transfer and the effect of radiation on convective heat transfer in the cooling of a laminar flow of gray medium in a short cylindrical channel were investigated. The energy equation was solved numerically in a wide range of characteristic parameters with radial and longitudinal radiant fluxes taken into account.

Several studies of combined heat transfer in channels have been made in recent years [1-7], but the problem of the interaction of heat conduction, convection, and radiation in combined heat transfer has not yet been adequately investigated. The existing theoretical investigations usually contain rather drastic simplifying assumptions, which are introduced in the calculation of the divergence of the radiant flux. In particular, in calculations of the radiative transfer in combined heat transfer the axial temperature distribution in the channel and the presence of axial radiant fluxes are usually ignored [1-6]. An exception is [7], where the case of heating of carbon dioxide in the entrance region of a round tube was investigated.

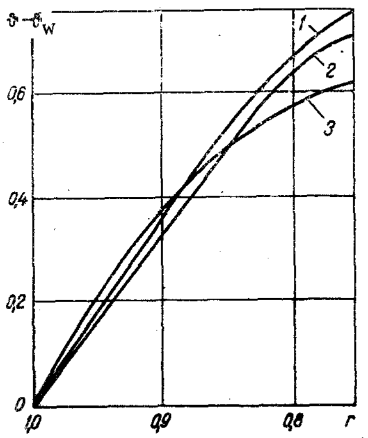

Fig. 1

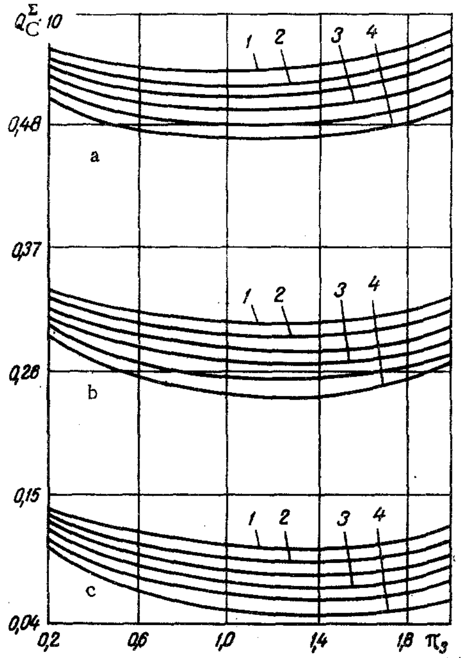

Fig. 2

Fig. 1. Radial temperature distribution near wall for different values of π_3 and x = 1.5, L_C = 18, π_1 = 0.034, Pe = 1000: 1) π_3 = 0; 2) 0.25; 3) 4.0.

Fig. 2. Dependence of Q_C^Σ on π_1, π_3, and ϑ_W for Pe = 1000, L_C = 12: a) ϑ_W = 0.2; b) 0.5; c) 0.8; 1) π_1 = 0.01; 2) 0.02; 3) 0.04; 4) 0.06.

All-Union Scientific-Research Institute of Metallurgical Heat Engineering, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 26, No. 2, pp. 238-244, February, 1974. Original article submitted July 25, 1973.

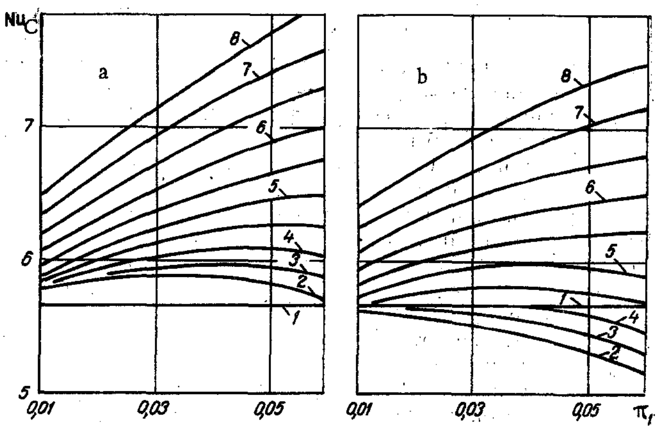

Fig. 3. Dependence of Nu_C against π_1, π_3, and ϑ_w for Pe = 1000, x = 6, L_C = 12: a) ϑ_w = 0.2; b) 0.6; 1) π_3 = = 0; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1.0; 6) 1.4; 7) 1.8; 8)2.0.

In the present work we investigate the interaction of radiation and convection in the case of combined heat transfer in a short round cylindrical channel through which flows a gray, radiating and absorbing, nonscattering medium. The side walls of the channel are black and have a constant temperature throughout. The entrance and exit ends of the channel are closed by porous black disks with constant, arbitrarily assigned, temperatures. The gas temperature at the entrance to the channel is constant over the cross section and can differ from that of the entrance disk. The flow of the medium throughout the length of the channel is steady and laminar. The case of cooling of the flow is considered.

The energy equations for an elementary volume with the longitudinal heat conduction of the medium ignored and the boundary conditions in dimensionless form are

$$\varphi \frac{\partial \vartheta}{\partial x} = \frac{4}{\mathrm{Pe}\, r} \frac{\partial}{\partial r}\left(\lambda r \frac{\partial \vartheta}{\partial r}\right) - \pi_1 \pi_3 (4\vartheta^4 - \eta_{\mathrm{inc}}); \tag{1}$$

$$x = 0;\ 0 \leqslant r \leqslant 1;\ \vartheta = 1; \tag{2}$$

$$x > 0;\ r = 1;\ \vartheta = \vartheta_w; \tag{3}$$

$$x > 0;\ r = 0;\ \left(\frac{\partial \vartheta}{\partial r}\right)_{r=0} = 0. \tag{4}$$

The dimensionless bulk density of the incident radiation at a given point in the medium is given by the expression

$$\eta_{\mathrm{inc}} = 2 \int_{-\pi/2}^{\pi/2} \int_{0}^{\pi} \left[B_w \exp(-\pi_3 S_N) + \frac{\pi_3}{\pi} \int_{0}^{S_N} \vartheta^4(s) \exp(-\pi_3 s)\, ds \right] \sin\theta\, d\theta\, d\psi. \tag{5}$$

Taking (5) into account we solved equation (1) numerically with boundary conditions (2)-(4), using a finite-difference method based on a implicit six-point symmetric scheme, and the pivot method in conjunction with the method of successive approximations [12]. As the zero approximation we used the temperature field obtained on solution of the problem of convective heat transfer in the entrance region of the tube. To solve equation (1) we used a regular orthogonal grid. To select the number of intervals between the nodes on the radius we compared the results when the radius was divided into 8, 16, and 32 intervals. The main calculations were made with the radius divided into 16 parts. To save machine time we calculated expressions (5) and (8) at the nodes of an irregular grid. The number of intervals on the radius in this case was five, and the number of intervals on the length was six. The values at the remaining nodes, which are required for the solution of equation (1), were found by the method of quadratic interpolation. When the results of an iteration were used directly to calculate the next iteration the convergence was poor, and in some cases the results even diverged. To obtain more rapid convergence in the calculation of the temperature fields we used the lower relaxation method [11]. We took the parameter as 0.5. This shortened the calculation when the changes in the relative temperatures ϑ in the iterations were less than one per cent. This usually required 4-6 iterations. To reduce the number of iterations in several cases we obtained initial conditions by solving problems in which the values of the characteristic parameters were close to the considered value. As a result of the solution of equation (1) we obtained the temperature distribution in the volume of the channel in the case of combined heat transfer, which depends on the main parameters of the problem:

$$\vartheta = \vartheta(r;\ x;\ \pi_3;\ \pi_1;\ \text{Pe};\ \vartheta_W;\ L_C). \tag{6}$$

The results discussed in the present work were obtained for channels with a length of 12 and 18 diameters.

The investigation of the distribution of axial radiant fluxes over the length of the channel showed that their value changed significantly only in regions of length 1.5 D-3.0 D at the ends of the channel. Over the whole remaining length their value was practically constant and, hence, had no effect on heat transfer by radiation between the gas volumes.

From the obtained temperature distribution (1) we found the longitudinal distribution of the local values of the mean-flow temperature, the convective heat flux to the wall Q_C, and the heat flux conveyed to the wall by radiation Q_R:

$$Q_C = \frac{2}{\text{Pe}}\left(\frac{\partial\vartheta}{\partial r}\right)_{r=1}, \tag{7}$$

$$Q_R = \pi_1\left\{2\int\limits_{-\pi/2}^{\pi/2}\int\limits_{0}^{\pi/2}\left[B_W \exp(-\pi_3 S_N) + \frac{\pi_3}{\pi}\int\limits_0^{S_N}\vartheta^4(s)\exp(-\pi_3 s)\,ds\right]\right.$$
$$\left.\times\cos\theta\sin\theta d\theta d\psi - \pi B_W(x)\right\}. \tag{8}$$

The total heat fluxes to the side wall of the whole channel due to radiation Q_R^Σ, convection Q_C^Σ, and the total resultant flux Q_C^Σ were found by integration of the corresponding local values over the length of the channel.

The error of the results $\text{Nu}_C = \text{Pe}\ Q_C/\bar{\vartheta}-\vartheta_W$ in the numerical solution of the problem of convective heat transfer in the entrance region of the tube (laminar flow) in accordance with the algorithm used was less than 1% when $x > 0.5$. A comparison was made with the exact solution [9]. The accuracy of the algorithm used to calculate the resultant radiant fluxes in the volume and on the walls was checked by comparing the emissivity of an infinite cylinder with cold black walls, filled with a gray medium, for values of π_3 up to 7.0, with the exact values given in [10]. The error was less than 1.5%.

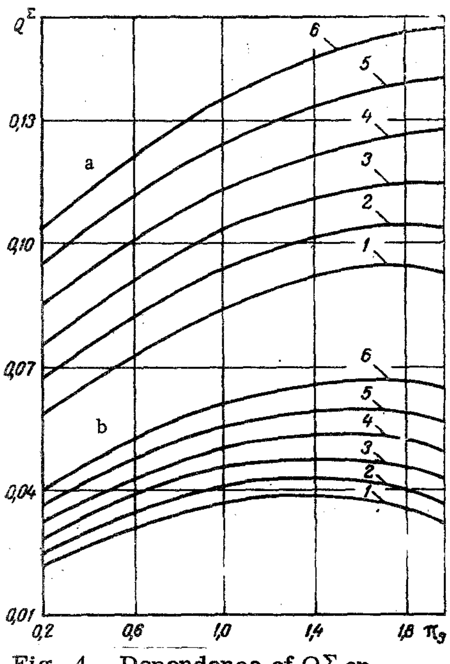

Fig. 4. Dependence of Q^Σ on π_3, π_1, and ϑ_W for Pe = 1000, L_C = 12: a) ϑ_W = 0.2; b) ϑ_W = 0.8; 1) π_1 = 0.01; 2) 0.02; 3) 0.03; 4) 0.04; 5) 0.05; 6) 0.06.

To obtain the relationships shown in Figs. 2-4 we used the method of orthogonal compositional experimental design [8]. The variable factors were π_1, π_3, and ϑ_W. The error of the empirical relations obtained by treatment of the results of our calculated variants by the method of [8] was as follows: for the mean-flow temperature $\bar{\vartheta}$ less than 4.5%; for Q_C^Σ and Nu_C less than 7%; and for Q^Σ less than 10%.

The conducted calculations showed that the dependences of the local and resultant radiative heat transfer on the wall temperature have a maximum. The position of the maximum depends on the specific conditions, i.e., on π_1, π_3, Pe, and L_C. For instance, when π_3 = 0.5, π_1 = 0.034, Pe = 2000, and L_C = 18 the maximum radiative heat transfer was obtained when $\vartheta_W \approx 0.4$. With change in the Bouguer number from 0 to 4.0 the total radiant flux Q_R^Σ also passed through a maximum in the region of π_3 = 1.0-2.0. The precise position of the maximum depends on the values of the other parameters (π_1, Pe, ϑ_W, L_C).

The dependences of the local convective heat fluxes Q_C in each cross section of the channel on π_3 have a minimum. The closer the section to the entrance, the lower the value of π_3 at which the minimum occurs. We reported similar relationships in a slotlike channel [1]. They can be attributed to the different nature of the radiant interaction between the hot core of the cooled flow and the colder layers at the wall in a weakly absorbing and strongly absorbing medium. Figure 1 shows the transverse distribution of temperature of the medium near the side wall for $\pi_3 = 0$, 0.25, and 4.0. Figure 1 shows that in a weakly absorbing medium an increase in the Bouguer number leads to a more rapid cooling of the wall layers of the stream than in a diathermal medium, whereas in a strongly absorbing medium the cooling of the wall layers is retarded. The intensive radiative interaction of the different volumes of medium leads to a leveling out of the transverse temperature field and to an increase in the transverse temperature gradient at the wall with increase in the Bouguer number.

The total convective heat flux to the channel wall also passes through a minimum with increase in the Bouguer number. Figure 2 shows Q_C^Σ as functions of π_3 and π_1 at different wall temperatures. As Fig. 2 shows, the relative effect of π_3 and π_1 on Q_C is more pronounced at high wall temperature, when the contribution of convective heat transfer is reduced. The conducted calculations showed that the effect of the Bouguer number on Q_C and Q_C^Σ is more pronounced, the greater the value of the Peclet number. An increase in Pe from 500 to 2000 leads to a reduction of the contribution of convective heat transfer by 50-60%, while the contribution of radiative heat transfer increases by 30-50%. The effect of π_1 and π_3 on the convective heat transfer coefficient (Nu_C) at different wall temperatures is illustrated in Fig. 3, which shows that in combined heat transfer the convective heat transfer coefficient is not a function of the Peclet number alone, but depends significantly on all conditions (π_1, π_3, ϑ_W, L_C). Our calculations showed that in conditions where Pe = 1000, $\vartheta_W = 0.2$-0.8, $L_C = 12$, and $\pi_1 = 0.01$ the local values of Nu_C change by 6-20% when π_3 increases from 0 to 2.0. In the same conditions with $\pi_1 = 0.06$ the local values of Nu_C change by 15-40%. It should be noted that the effect of π_3 on Nu_C is greater at larger Pe values.

At low wall temperatures with increase in the Bouguer number from 0 to 2.0 in channel sections close to the entrance Nu_C passes through a minimum, which lies in the region of small values of π_3 (~0.2). With increase in wall temperature this feature is observed also in channel sections far from the entrance. For instance, Fig. 3 shows that with $\vartheta_W = 0.6$ in cross section x = 6 the convective heat transfer coefficient in an absorbing medium for $\pi_3 = 0.2$, 0.4, and 0.6 has lower values than in a diathermal medium in the same conditions. The effect of the main parameters of the problem on the total radiative-convective heat transfer is of interest. Figure 4 shows the dependence of the total heat flux transmitted to the side wall of the channel by convection and radiation on π_3, π_1, and ϑ_W (for Pe = 1000; $L_C = 12$). This figure shows that when the Bouguer number changes from 0.2 to 2.0 the total heat transfer passes through a maximum whose position on the π_3 axis depends on the other parameters of the problem. Our calculations showed that the total heat transfer (convection + radiation) coefficient, like the total resultant heat flux on the side wall, depends significantly on all the main parameters of the considered problem.

NOTATION

$\varphi = w/\bar{w}$	is the velocity at given point of channel relative to mean-flow velocity $\bar{w}$ over cross section of channel;
$\vartheta = T/T_0$	is the dimensionless temperature relative to stream temperature T_0 at entrance to channel;
ϑ_W	is the dimensionless wall temperature;
$\bar{\vartheta}$	is the dimensionless mean-flow temperature of stream at given cross section x;
$x = x_c/D$ and $r = r_c/R$	are the dimensionless longitudinal and radial coordinates, respectively;
$Pe = \bar{w}Dc_p\rho/\lambda$	is the Peclet number;
$\pi_1 = \sigma_0 T_0^3/\bar{w}c_p\rho$	is the reciprocal of Boltzmann number;
c_p, ρ and λ	are the specific heat, density, and thermal conductivity of medium, respectively, independent of temperature;
$\pi_3 = kD$	is the Bouguer number;
K	is the absorption coefficient of gas;
$\eta_{inc} = \frac{1}{\sigma_0 T_0^4}\int_{4\pi} B d\omega$	is the dimensionless incident bulk radiation density at given point;
B_W	is the brightness of effective emmision of wall relative to $\sigma_0 T_0^4$;
Q_R and Q_C	are the dimensionless local resultant heat fluxes due to radiation and convection, respectively, in relation to $(\bar{w}c_p\rho T_0)$;
Q_C^Σ, Q_R^Σ, and $Q^\Sigma = Q_C^\Sigma + Q_R^\Sigma$	are the dimensionless total convective, resultant radiative, and total heat flux onto side wall of channel;

S_N	is the distance from considered point to wall of channel in direction of beam S in relation to D;
$d\omega$	is the elementary solid angle;
$D = 2R$	is the channel diameter;
L_C	is the length of channel in relation to D;
Nu_C	is the local Nusselt number calculated from Q_C.

LITERATURE CITED

1. V. N. Timofeev, F. R. Shklyar, et al., in: Collection of Scientific Works of All-Union Scientific-Research Institute of Metallurgical Heat Engineering [in Russian], No. 13, Metallurgiya, Moscow (1967).
2. R. Viskanta, J. Heat Transfer, 85, 318 (1963).
3. E. Pearce and A. F. Emery, J. Heat Transfer, 92, 221 (1970).
4. I. Habib and R. Grief, Int. J. Heat and Mass Transfer, 13, No. 10, 1571 (1970).
5. C. S. Landram, R. Grief, and I. S. Habib, J. Heat Transfer, 91, 330 (1969).
6. V. N. Adrianov, Fundamentals of Radiative and Combined Heat Transfer [in Russian], Énergiya, Moscow (1972).
7. S. Desoto, Int. J. Heat and Mass Transfer, 11, No. 1, 39 (1968).
8. Yu. P. Adler, Kh. G. Gurevich, and A. G. Shvarts, in: Design of Experiments and Use of Computer Technology in the Synthesis of Rubber [in Russian], Khimiya, Moscow (1970).
9. B. S. Petukhov, Heat Transfer and Resistance in Laminar Flow of Liquid in Tubes [in Russian], Énergiya, Moscow (1967).
10. A. S. Nevskii, Radiative Heat Transfer in Ovens and Furnaces [in Russian], Metallurgiya (1971).
11. A. D. Gosman et al., in: Numerical Methods of Investigating Viscous Flows [Russian translation], Mir, Moscow (1972).
12. V. K. Saul'ev, Integration of Equations of Parabolic Type by the Grid Method [in Russian], Fizmatgiz Moscow (1960).